An application of business activity modeling to regional production and national distribution of plywood

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Abstract

Markets for many of the products produced from wood, such as lumber, structural panels, particleboard, pulp, and paper, do not behave according to the economic assumptions of pure competition. Typical linear programming models are not well suited to modeling markets for quantitative decision analysis because many of the individual firms are large enough to impact market price. A new modeling technique, Business Activity Modeling, which is based on components of demand analysis, linear programming, and reactive programming, was tested on the market for structural plywood. The model allows a producer to quantitatively determine optimum production and distribution levels for spatially separated markets. The analysis effectively simulated regional production and national distribution of plywood for a hypothetical company. The modeling technique may also be suitable for petroleum products, steel, aluminum, or any market not characterized as purely competitive.

Linear programming models (LPM) are commonly used in many industries to maximize or minimize some quantity (usually profits or costs) (Anderson et al. 2000) subject to a set of linear constraints. However, typical LPMs (Nemhauser et al. 1991) are not capable of maximizing profits when firms are large enough to influence market price. Reactive programming (Tramel and Seale 1959), which incorporates demand analysis, is a superior technique for evaluating the competitive position of producing areas and determining equilibrium distribution patterns. However, neither of these procedures are well suited to evaluating options of companies that produce multiple products and that are large enough to influence market price. A third technique, business activity modeling (BAM), is based on components of demand analysis (Ferguson and Gould 1975), linear programming (Hillier and Lieberman 1980), and reactive programming. BAM is oriented toward companies large enough to impact market price and allows determination of maximum profit production and distribution levels for markets that are geographically separated. The example presented in this paper illustrates regional plywood production with national plywood distribution; however, the technique and theory apply to lumber, other structural panels, particleboard, gypsum products, pulp, paper, petroleum products, steel, aluminum, air fares, or any market not characterized as perfectly competitive.

Economic rationale

Markets that are studied by economists are classified as purely competitive, oligopoly, monopolistic competition, or pure monopoly. The distinguishing difference among markets is determined by the number of buyers and sellers engaged in commerce. The total number of buyers and sellers determines the impact that any one individual or firm would have on the market from a supply or demand perspective. The most commonly used model in analyses is pure competition which assumes many

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buyers and sellers so individual firms can have no impact on the price in the market. In the other models, buyers or sellers may impact market price to some degree or even be able to control it in the case of a monopolist. The classification of a market into one of the models changes the slope of the supply curve or demand curve that is used in analysis. Economic models suggest that firms should produce where marginal cost is equal to marginal revenue (Awh 1976, Kamerschen and Valentine 1977), regardless of the market type.

One of the most commonly used quantitative techniques used to model economic behavior is linear programming. The assumptions of linear programming are consistent with the economic model of pure competition. Linear programming modelers usually maximize by substituting product price for marginal revenue. This procedure is appropriate and yields excellent solutions for purely competitive markets because price is equal to marginal revenue; however, only in pure or perfect competition is marginal revenue equal to price. If the market price is impacted by the corporation as a whole, the demand curve is downward sloping and marginal revenue is no longer equal to price (Doll et al. 1968). In response, linear programming modelers typically block the objective function by allowing quantity *X* to be sold for one price and quantity *Y* to be sold for another (usually lower) price. While this method attempts to simulate a downward sloping demand curve, the model is unsatisfactory because the appropriate economic criterion necessary for profit maximization (marginal revenue = marginal cost) is not satisfied.

Business activity modeling

Business activity modeling (BAM) is a quantitative technique that is consistent with all four models of economic theory. To correctly model a market, a demand function, a marginal revenue function, and a marginal cost function should be specified. The calculations require five basic bits of information:

- marginal cost,
- market share or market size,
- market price,
- firm quantity in the market, and
- price elasticity of demand.

The following example is for a hypothetical company producing plywood in nine southern states (regional production) and distributing to 52 market areas (national distribution). For simplicity, the theory presented in this paper illustrates a single product case, but the same relationships hold for many products and multiple inputs. The analysis in the latter part of the paper is based on a three product example.

Price elasticity of demand

Price elasticity of demand is the percentage change in quantity purchased for a corresponding percentage change in market price (Doll et al. 1968). **Figure 1** illustrates the relationship of demand, the slope of the demand curve, and the price elasticity of demand. In pure competition, the slope of the demand curve is zero (D1). Elasticity and slope are inversely related. A perfectly inelastic demand curve (D2) would have zero price elasticity and an infinite slope. Price elasticity of demand changes for each price and quantity relationship of D3. When MR3 is positive, the price elasticity of demand is greater than one or elastic. When MR3 is zero, price elasticity of demand is equal to one or unitary. When MR3 is negative, price elasticity of demand is negative. Since a firm should always equate marginal revenue with marginal cost, profit maximization will always occur in the elastic portion of the demand curve if costs are positive. Several empirical studies using historical cross-sectional time series data indicate that the demand for plywood is inelastic on a national basis. However, it is possible for a firm to produce in the elastic range of the firm demand curve while the market demand curve is in the inelastic range.

Average production cost and limitations

The example includes nine hypothetical plants in states across the South. **Figure 2** illustrates hypothetical plant locations and average total cost of production assumed at each plant. The numbers listed in each state from top to bottom represent sanded, specialty, and sheathing plywood average total costs of production. The cost numbers calculated represent costs of the industry in general and simulate regional differences in timber prices.

Each hypothetical plant had production capacity of 6 million ft.^{2}, 3/8-inch basis, per week. The initial product mix was 25 percent sanded, 25 percent specialty, and 50 percent sheathing. The basic strategy limited sanded production to 1.5 million ft.^{2}, 3/8-inch basis, at each plant to reflect grade production limitations caused by raw material quality. Any unused sanded production could be put into specialty production. The specialty production limit was 1.5 million ft.^{2}, 3/8-inch basis, plus any unused sanded production with the total not to exceed 3 million ft.^{2}, 3/8-inch basis. Finally, the sheathing production limit was not to ex-

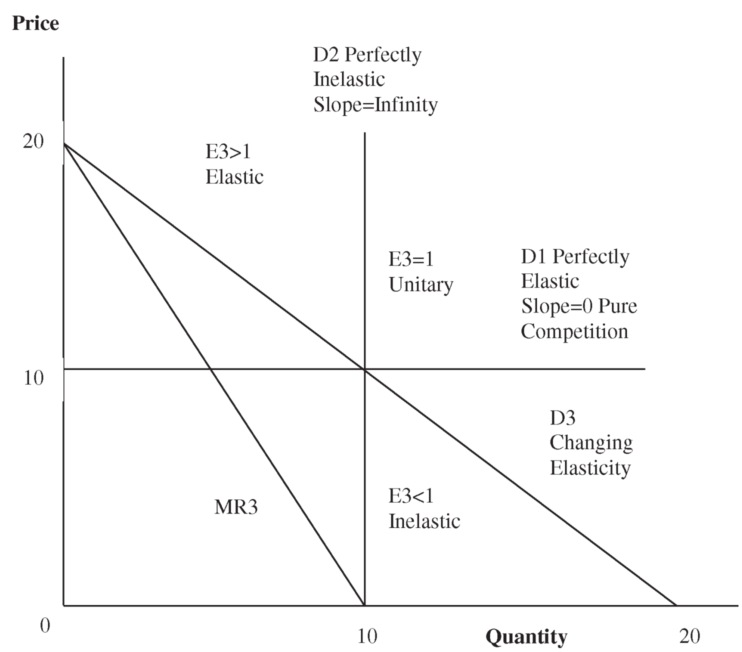

Figure 1. — Relationship of price elasticity of demand and slope of demand function.

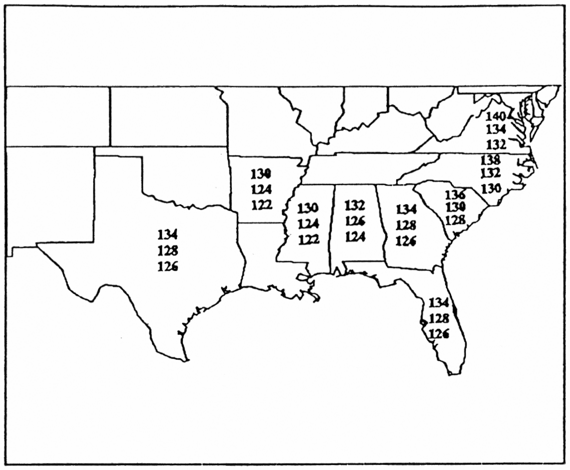

Figure 2. — Plant location and cost of production for sanded, specialty, and sheathing plywood, respectively.

Table 1. — Average weekly production of southern plants.[a]

Plywood type	Actual	Nine plants	% of actual
	- - - - - (1,000 in.^{2}, 3/8-in. basis) - - - - -		(%)
Sanded	17,086.00	13,500.00	79.0
Specialty	16,459.00	13,500.00	82.0
Sheathing	157,351.00	27,000.00	17.2
Total	190,896.00	54,000.00	23.8

[a] Source: Anderson 1991, p. 34. Total southern shipments reported weekly.

Table 2. — FOB mill prices by region of the United States.[a]

Plywood type	Western	Central	Eastern
	- - - - - - - - - - - ($/1,000 ft.^{2}, 3/8-in. basis) - - - - - - - - - - -		
Sanded	205	205	213
Specialty	175	168	169
Sheathing	152	149	153

[a] Source: Random Lengths, selected issues.

ceed 6 million ft.^{2}, 3/8-inch basis, including any unused sanded and specialty production capacity.

Shipping costs

The estimated mileage from each plant location to each market was obtained from a road atlas (Rand McNally 1991). The assumed transportation rate from each plant to each market was $0.03 per M[1] per mile for sanded products and 0.025 per M per mile for specialty and sheathing products. The transportation rate for sanded products was assumed to be higher because sanded products are heavier. A marginal cost function for each plant in each market was based on the assumed production costs, transportation mileage matrix, and transportation rates. Shipping costs were not included for export, Alaska, and Honolulu since mileage figures would bias actual water delivery rates.

Market share

Industry distribution data was used to provide data for the hypothetical company. The nine hypothetical plants had a share in each market equal to the proportion of southern pine plywood in that market times their combined percentage of southern production and distribution. Average weekly shipments by product to each of the 52 market areas (Anderson 1991, p. 28) and shipments of southern plants to each of the 52 markets provided the basis for calculation of market share for each southern product in each market (Anderson 1991, p. 34). The actual southern production of sanded, specialty, and sheathing plywood is listed in **Table 1** along with the assumed production and production percentage of the nine hypothetical plants.

Average weekly shipments of southern plants, southern market share by product, and the data from **Table 1** resulted in a hypothetical company market share for each market. For example, the Atlanta-Chattanooga sanded market had 1,836 ft.^{2}, 3/8-inch basis, per week. The southern share of this market was 64.85 percent or $1,836 \times 0.6485 = 1,191$ ft.^{2}, 3/8-inch basis. The company market share was $1,191 \times 0.79 = 941$ ft.^{2}, 3/8-inch basis.

Sample market

Using Atlanta-Chattanooga as a sample market for sanded plywood, the average weekly total market quantity was 1,836 M. The assumed wholesale price, calculated using **Table 2** assuming a 30 percent markup, was $277/M ($213 × 1.3 = $277). Since price and quantity are known, one point on the real market demand curve can be absolutely identified. Either the size of the total market or the firm market share can be used to calculate volumes that other firms would deliver to the market. In this particular case, other firms would ship 895 units. Thus, the marginal revenue function would begin at point "b" in **Figure 3** because other firms expect our firm to ship 1,191 M. Our firm then equates marginal revenue with the appropriate marginal cost to determine the profit maximizing quantity and origin of shipment. The analysis calculates the marginal cost of each product in each market based on production costs plus freight. Two of the nine marginal cost functions in the Atlanta-Chattanooga market are illustrated in **Figure 3**. The marginal cost functions of delivered product from the Texas plant (MC-TX) and the Georgia plant (MC-GA) illustrate that each plant will have a different marginal cost function in each market. Using basic economic principals and assuming that changes in market quantities will be 10 percent or less, all other necessary information can be calculated given the five basic information bits. The BAM technique maximizes by making adjustments, in volume to selected markets and shipping routes.

[1] For simplification 1,000 ft.^{2}, 3/8-inch basis, will be referenced by the acronym M in this text.

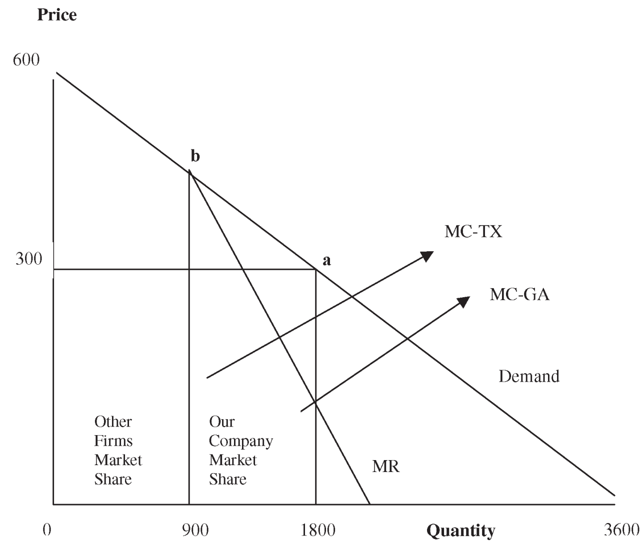

Figure 3. — Sample Atlanta-Chattanooga market with two of nine marginal cost functions. MC-TX is the Texas plant; MC-GA is the Georgia plant.

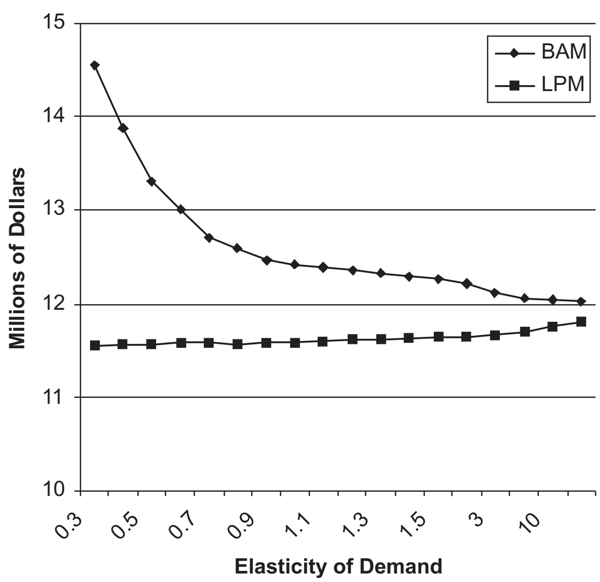

Figure 4. — Corporate sales by price elasticity of demand.

In **Figure 3**, price and quantity data precisely identify point *a* on the demand curve. Movements along the demand curve are strictly price related. However, demand is constantly shifting due to factors other than price. When a weekly data point of price and quantity is recorded, all of the factors that shift the demand function (interest rates, housing starts, building projects, etc.) are neatly captured and incorporated into the modeling system.

Procedure

BAM can utilize the price elasticity of the demand data by market; however, this example uses a wide range of elasticities and presents the results of many analyses assuming a constant price elasticity of demand for each product in each market. Finally, "reasonable" price elasticity of demand estimates for each product are identified, and the results of analyses are presented.

BAM is written in a form to estimate the true profit (marginal revenue = marginal cost) solution. Additionally, a switch can be set to run a marginal cost = P (linear programming) solution. When running in LPM mode, the model assumes that the objective function is perfectly blocked in unit increments to represent thousands of sales restrictions.

Production costs used in any quantitative modeling method should be marginal costs. Often, companies divide the total cost of production by units produced and used this unit cost in place of marginal cost. This unit cost is actually (in economic terms) average total cost (ATC) and is not the appropriate decision criteria. In practical terms, if a plant manager schedules overtime to increase production, the overtime makes the additional units cost more. This cost structure is difficult to model in LPM, but BAM can easily accommodate this type of structure. In the following example when running in LPM mode, marginal cost of each plant fixed at levels depicted in **Figure 3** less $12.50 per M for fixed cost. However, in BAM analyses the marginal cost function was specified so that at full production the ATC of each product was equal to costs indicated in **Figure 2**. Fixed costs in both models were assumed to be $75,000 per week to yield an average fixed cost (AFC) of $12.50 per M (75,000 per 6,000). The marginal cost functions for each plant are listed in **Table 3** along with values of ATC, AFC, and average variable cost (AVC) for each product at full production.

Results

Projected corporate sales based on the recommendations of each type of model are illustrated in **Figure 4**. As price elasticity of demand increases, the slope of the demand curves decreases, and the differences between the two techniques decrease. As elasticity of demand approaches infinity, the two management plans will converge. This result is consistent with the theory described in **Figure 1**. The differences in weekly profits associated with the two management models are contrasted in **Figure 5** which is a summary of 18 different analyses of BAM and LPM. In the LPM solution, profits increase as the price elasticity of demand approaches infinity. Only in the perfectly elastic (zero slope) market can the LPM approach recommend market quantities that are consistent with profit maximization. **Figure 6** shows the product mix of each model as the price elasticity of demand varies. In highly inelastic markets, the BAM model recommends reduced production for all items. The sum of the product mix shown in **Figure 6** is illustrated in **Figure 7** by the price elasticity of demand. In the very inelastic range, between –0.3 and –0.6,

Table 3. — Cost functions of three plywood types used in BAM analyses.[a]

	Production	AFC	AVC	ATC	MC-intercept	MC-slope	MC
Sanded							
Georgia	1,500	12.50	121.5	134	116.27	0.007	126.77
Arkansas	1,500	12.50	117.5	130	112.27	0.007	122.77
Mississippi	1,500	12.50	117.5	130	112.27	0.007	122.77
Alabama	1,500	12.50	119.5	132	114.27	0.007	124.77
Florida	1,500	12.50	121.5	134	116.27	0.007	126.77
South Carolina	1,500	12.50	123.5	136	118.27	0.007	128.77
North Carolina	1,500	12.50	125.5	138	120.27	0.007	130.77
Virginia	1,500	12.50	127.5	140	122.27	0.007	132.77
Texas	1,500	12.50	121.5	134	116.27	0.007	126.77
Specialty							
	Production	AFC	AVC	ATC	MC-intercept	MC-slope	MC
Georgia	1,500	12.50	115.5	128	111.02	0.006	120.02
Arkansas	1,500	12.50	111.5	124	107.02	0.006	116.02
Mississippi	1,500	12.50	111.5	124	107.02	0.006	116.02
Alabama	1,500	12.50	113.5	126	109.02	0.006	118.02
Florida	1,500	12.50	115.5	128	111.02	0.006	120.02
South Carolina	1,500	12.50	117.5	130	113.02	0.006	122.02
North Carolina	1,500	12.50	120.5	132	115.02	0.006	124.02
Virginia	1,500	12.50	122.5	134	117.02	0.006	126.02
Texas	1,500	12.50	115.5	128	111.02	0.006	120.02
Sheathing							
	Production	AFC	AVC	ATC	MC-intercept	MC-slope	MC
Georgia	3,000	12.50	113.5	126	106.02	0.005	121.02
Arkansas	3,000	12.50	109.5	122	102.02	0.005	117.02
Mississippi	3,000	12.50	109.5	122	102.02	0.005	117.02
Alabama	3,000	12.50	111.5	124	104.02	0.005	119.02
Florida	3,000	12.50	113.5	126	106.02	0.005	121.02
South Carolina	3,000	12.50	115.5	128	108.02	0.005	123.02
North Carolina	3,000	12.50	117.5	130	110.02	0.005	125.02
Virginia	3,000	12.50	120.5	132	112.02	0.005	127.02
Texas	3,000	12.50	113.5	126	106.02	0.005	121.02

[a] AFC = average fixed cost; AVC = average variable cost; ATC = average total cost; MC = marginal cost.

the BAM model suggests less total production than suggested by the LPM. Total production costs by price elasticity of demand (**Fig. 8**) are consistent with the shape of the total production curve.

Figure 9 shows the total shipping costs associated with each modeling technique. The LPM solution would result in lower shipping costs than the BAM solution except at very low elasticities (less than –0.3) where BAM recommends reduced total product shipments. Since LPM suggests more product in each market, the closer markets will receive more product, which will reduce freight costs. However, the BAM solution recognizes that reduced quantities in some markets mean higher profits and will suggest less products in each market but will service more total markets. Since more markets are suggested

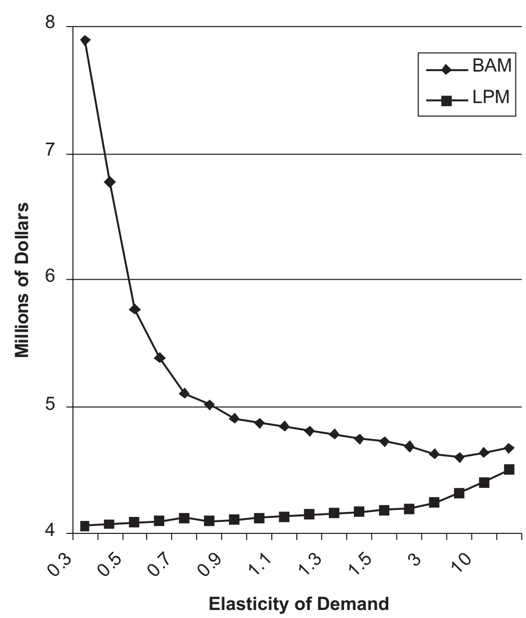

Figure 5. — Corporate profits by price elasticity of demand.

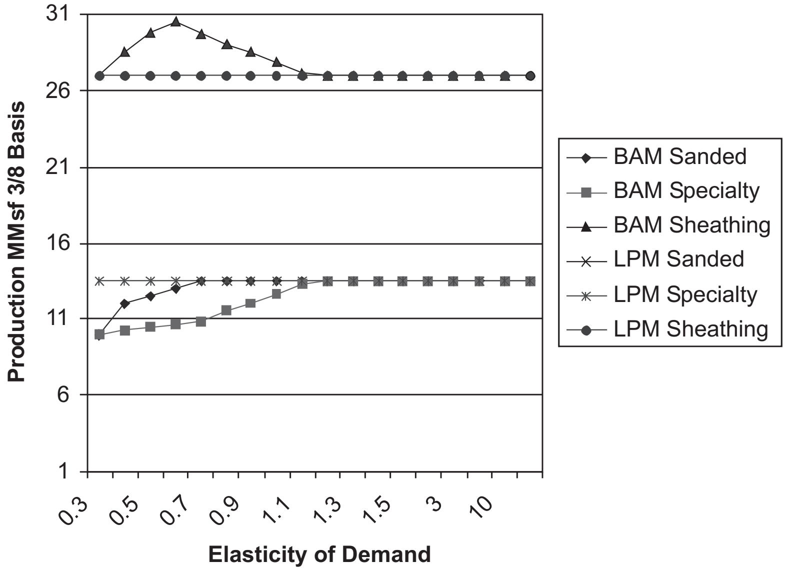

Figure 6. — Product mix and production by price elasticity of demand. Data overlap for LPM sanded and specialty production.

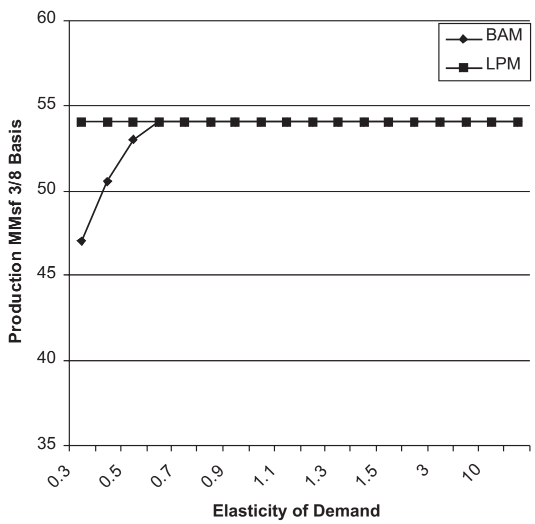

Figure 7. — Total production by price elasticity of demand.

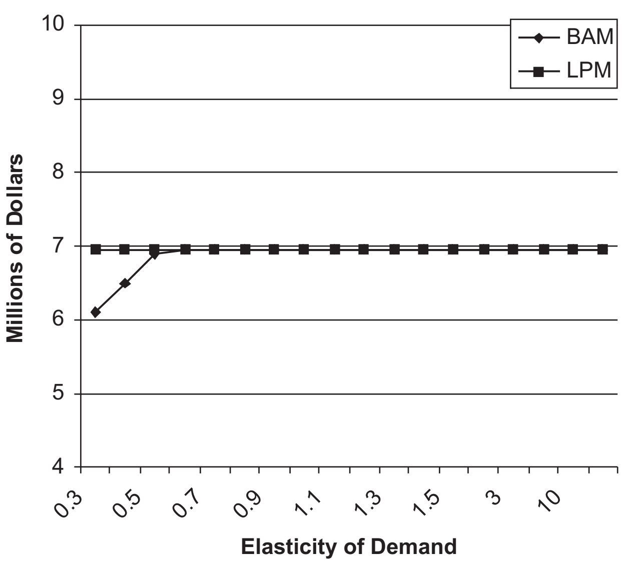

Figure 8. — Corporate total cost of production by price elasticity of demand.

for distribution (most further away than LPM), shipping costs increase. BAM also minimizes the cost of delivered goods for selected markets quantities and distribution patterns. The increases in sales shown in **Figure 4** exceed the increased shipping costs depicted in **Figure 9** resulting in the increased profits illustrated by **Figure 5**.

Although BAM offers advantages for various price elasticities of demand, companies cannot choose what price elasticity of demand suits them best and must simply adapt to the market. Empirical studies suggest that the price elasticity of demand for plywood is highly inelastic. This analysis indicates that the price elasticity of demand probably varies for each product and each market. Production is not fully allocated to markets using BAM analyses if the initial price elasticity of demands falls below –0.8 for sanded, –1.4 for specialty, and for –0.6 for sheathing. This conclusion is consistent with previous studies projecting inelastic markets, but estimates are higher than analyses of cross-section time-series data indicate. The estimates presented here are consistent with microeconomic theory and appear reasonable. The specialty elasticity of –1.4 is not as surprising as it seems. Since firms will always produce in the elastic portion of the demand curve and the example nine plants account for a large percentage of total specialty production (46% of specialty as opposed to 28% of sanded and 11% of sheathing) , this estimate is also consistent with microeconomic theory. Given elasticity estimates which allow full production and distributions, calculated slopes for each product in each market indicate flatter slopes are present in the larger markets. For example, the Atlanta-Chattanooga market would be impacted less than the Birmingham market if equal quantities of the product were shipped to each because Birmingham is a smaller market.

The difference in the corporate profits between BAM and LPM is approximately $487,000 per week or $25 million per year for the corporation. Shipping costs increase $3.67/M or $203,000 per week (an annualized rate of $10.5 million). Mill returns increase $6.94/M or $375,000 per week ($19.5 million per year) assuming constant demand functions and costs. The LPM individual mill returns are approximately equal, whereas the BAM individual mill returns are significantly different. These

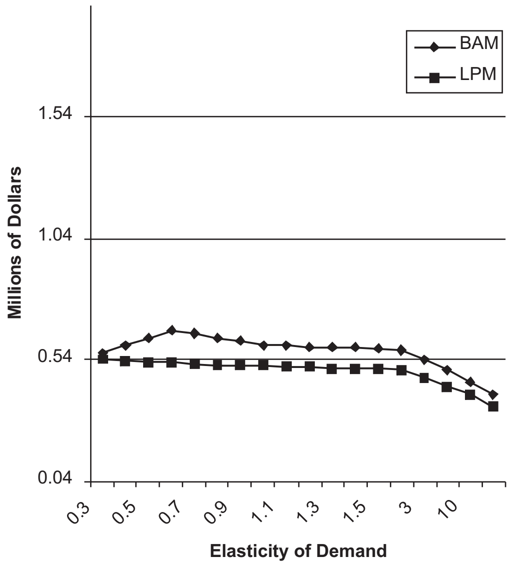

Figure 9. — Corporate shipping costs by price elasticity of demand.

observations suggest that equalizing the mill returns of individual plants will not lead to profit maximization. Of course, these differences are the product of management recommendations from quantitative procedures which, in reality, are almost never followed precisely. However with additional sales restrictions, LPM could improve but could not exceed BAM performance levels. At best, LPM might equal BAM performance because BAM equates marginal revenue with marginal cost in all markets for all products.

Activity adjustment for catastrophes

BAM is a powerful management tool in short-term emergency situations. When management is faced with lost production from a labor strike, fire, storm, or other calamity at one or more plants, alternative production and distribution plans must be developed on short notice. If BAM is in place, management can have the most profitable adjusted production and distribution plan within a matter of minutes.

Computer requirements

The BAM modeling system will work with a wide variety of computing platforms including personal computers. The computer must have an editing program or Microsoft Access database software and the ability to run FORTRAN executable programs.

Summary and conclusions

Analyses of modeling regional production and national distribution of plywood were studied for business activity programming and compared to linear programming. A theory of model behavior was also presented. BAM is superior to LPM whenever the market is not perfectly elastic and whenever the marginal cost function has a positive slope. The difference between the two modeling systems diminishes as price elasticity of demand approaches infinity and the slope of MC approaches zero.

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